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Four new papers were published or in preparation during the past year. The results are in four main areas: (1) Frequency domain tests for optimality in problems of periodic control, (2) Infinite-horizon problems as natural extension of traditional problems in periodic control, (3) Applications of periodic control to aircraft cruise, and (4) Minimal realization of nonlinear (functional 2-power) input/output maps.

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Interim Scientific Report

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SYSTEM OPTIMIZATION BY PERIOIDIC CONTROL

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Department of Aerospace Engineering

The University of Michigan

Period: October 1, 1978 through September 30, 1979

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This report covers research which was carried out from October 1, 1978 through September 30, 1979 under Grant No. AFOSR 77-3158. The results are in four main areas: (1) Frequency domain tests for optimality in problems of periodic control, (2) Infinite-horizon problems as a natural extension of traditional problems in perioidic control, (3) Applications of periodic control to aircraft cruise, (4) Minimal realization of nonlinear (functional 2-power) input/output maps. The attached bibliography includes: items which have been reported in previous Interim Scientific Reports and are required for reference here [1, 2, 5], new items [3, 6], and reports currently in preparation [4, 7]. The significance and interrelationship of these items and other new results is reviewed in the following paragraphs. Elmer G. Gilbert was the principal investigator. Daniel J. Lyons and Dennis S. Bernstein, doctoral students at The University of Michigan, also worked under the Grant and made important contributions.

The π test [a] is a second order test for optimality in periodic control problems. It involves sinusoidal perturbations in the neighborhood of a steady-state optimum and provides a powerful algebraic test which may determine whether or not a system is proper (time dependent periodic control gives better performance than steady-state control). Reports [1, 2] extended the work in [a] in three directions: (1) applied the π test to a more general periodic control problem, (2) gave conditions which assured the validity of the π test ([a] contained crucial omissions), (3) explored in detail relationships between second order conditions for optimality in the steady-state and dynamic problems.

During the last year the material has been extended further in two ways: the gap between necessary conditions and sufficient conditions for optimality has been closed (this was based on [b], whose applicability was discovered last fall), the proofs (contained in [2]) have been simplified. The extended results are now contained in a single report [3] which will appear as a regular paper in the December, 1979 issue of the IEEE Transactions on Automatic Control.

The preceding work assumes that the control input is unconstrained ($u(t) \in R^m$). A result which applies in the presence of constraints ($u(t) \in U \subset R^m$) has been obtained this year. It relies on a basic lemma in a recent paper by Warga [c] and provides a test for proper. It can be summarized somewhat imprecisely by some additions to the notation of [3]. Introduce

$$\hat{\pi} = \pi(0) + (\bar{C}G(0) + \bar{D})' \bar{H}_{yy}(\bar{C}G(0) + \bar{D}),$$

$$\tilde{\pi}(\omega) = \begin{bmatrix} (\hat{\pi} + \pi(\omega)) & (\hat{\pi} - \pi(\omega)) \\ (\hat{\pi} - \pi(\omega)) & (\hat{\pi} + \pi(\omega)) \end{bmatrix} \qquad \text{(2m x 2m matrix)}$$

Then the periodic control problem is proper if there exists a pair of vectors $v_1, v_2 \in U$ and an $\omega \geq \frac{2\pi}{T}$ such that

$$\bar{G}[\bar{C}G(0) + \bar{D}](v_1 + v_2) = 0, \qquad \bar{G} = \begin{bmatrix} \bar{g}_{oy} \\ \bar{M} \end{bmatrix}$$

and

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}' \tilde{\pi}(\omega) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} < 0 .$$

This "generalized π test" goes beyond the rather special case treated in [d]. Several examples have been examined which demonstrate that it is a useful test.

The new test requires a normality condition which is more complex than the condition given in [3]. Considerable effort was given to finding a more easily verified condition. It proved only partially successful. The present theory requires U to be convex, but it is clear that this condition may be relaxed in a variety of ways. The study of these and other related questions is now nearing completion.

In the usual periodic control problem the cost and constraints depend on a vector y (see [3] for notation) whose components are given by

$$y_i = \frac{1}{\tau} \int_0^{\tau} \tilde{f}_i(x(t),\ u(t))dt, \qquad i=1,\ldots,m,$$

where x(t), u(t) are (periodic) solutions of

$$\dot{x}(t) = f(x(t),\ u(t)), \quad x(0) = x(\tau).$$

When the period τ is large there is some question as to whether or not the formula for y_i is valid and the motion should be periodic. An alternative formulation is

$$y_i = \int_0^{\infty} \hat{f}(x(t), u(t))e^{-\delta_i t} dt, \qquad \delta_i > 0,$$

where $e^{-\delta_i t}$ represents the exponential discounting of future costs to the present. Some economists have considered problem formulations in this direction, but apparently without knowledge of results in periodic control (see, e.g., [e]). The theory of the above "infinite horizon" problem has many similarities with

the theory of periodic control. In particular, there is a steady-state optimization problem contained within the dynamic optimization problem: $x(t)$, $u(t)$ constant gives

$$f(x,u) = 0 \quad \text{and} \quad y_i = (\delta_i)^{-1} \hat{f}_i(x,u).$$

Thus it is possible to compare steady-state performance with dynamic performance. Preliminary investigations have produced some interesting results.

For example, if $x(t)$ and $u(t)$ are periodic with period τ (certainly a special choice for the infinite horizon problem), it can be shown that

$$y_i = (1 - e^{-\delta_i \tau})^{-1} \int_0^{\tau} e^{-\delta_i t} \hat{f}(x(t), u(t))dt.$$

For $\tau\delta_i << 1$, this gives

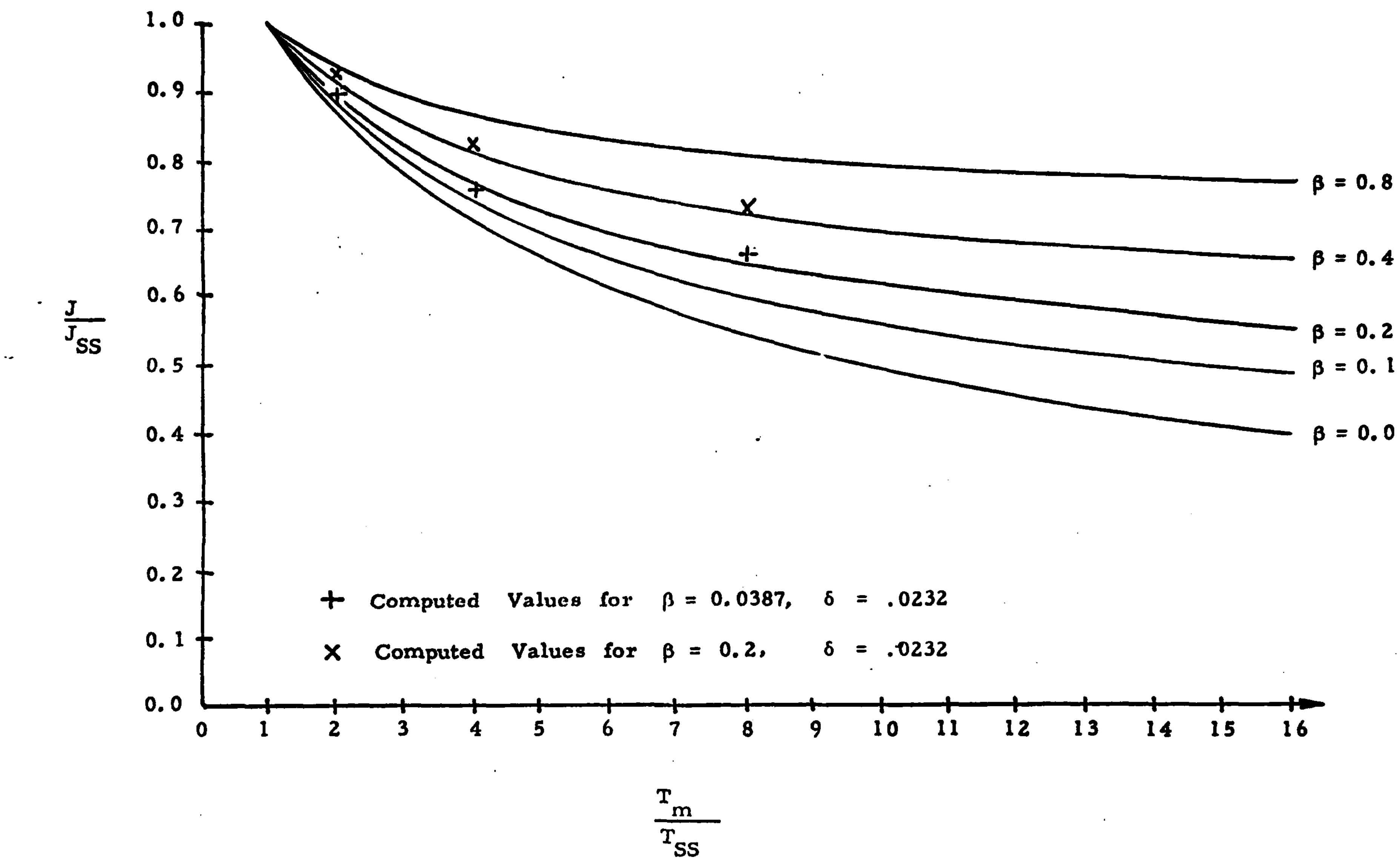
$$y_i \cong \frac{1}{\tau} \int_0^{\tau} (\delta_i)^{-1} \hat{f}(x(t), u(t))dt.$$

Thus, the usual periodic control problem can be viewed as an infinite horizon problem in which the motion is periodic, the period is small ($\delta_i\tau << 1$), and $\tilde{f}_i = (\delta_i)^{-1} \hat{f}_i$. Alternatively, the tools of periodic control theory may be applied to the study of the infinite horizon problem. This has been done with a variation of the π test and necessary conditions for the dynamic optimality of steady-state solutions have been obtained. This technique is also useful in examining steady-state solutions different than the one suggested above, e.g., those which appear when $x(0)$ is fixed (see [e]). Many open questions remain and will be the subject of research in the coming year.

The computational work on periodic cruise of aircraft was continued. The aircraft model used is described in [f] . Previous computations

concerned the optimization of specific range and assumed that σ (engine specific fuel consumption) was constant. This year a more realistic model for σ was considered. Its dependence on speed and altitude was a good representation of a typical small jet engine. The resulting optimal periodic motions were close to those obtained for σ = constant. With fairly strong constraints on the maximum altitude (say 20,000 ft. or less) the improvements in performance were notably better than for σ = constant. The poor steady-state cruise performance of the realistic engine at the constraint altitude was the principal factor in this difference.

In addition, the problem of maximizing endurance in cruise was investigated. The aircraft model was the one described in [f] with σ = constant. Analysis of the energy-state approximation gives a good indication of the principal factors which make periodic cruise more efficient than steady-state cruise. It shows that optimum periodic cruise tends toward low altitudes where maximum engine thrust is greater and flight speed (drag) is lower. Thus, it is necessary to impose an explicit constraint on minimum altitude ($h \geq h_o$). The figure shows the reduction in average fuel rate obtained by periodic motion: $J(J_{ss})^{-1}$ is the ratio of average fuel rate with periodic control to the optimal fuel rate with steady-state cruise, $T_m(T_{ss})^{-1}$ is the ratio of maximum engine thrust to the thrust required for maximum endurance cruise, $\beta = V_s^2(gh_o)^{-1}$ where V_s is the steady-state speed for maximum endurance. Optimum periodic performance using the full point-mass model gives the points designated by + and x. Several computations have been made using the realistic engine model. They show smaller



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$\frac{J}{J_{SS}}$
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
$\frac{T_m}{T_{SS}}$
β = 0.8
β = 0.4
β = 0.2
β = 0.1
β = 0.0
+ Computed Values for β = 0.0387, δ = .0232
× Computed Values for β = 0.2, δ = .0232

improvements than those indicated in the figure. However, if an altitude constraint of the form $h_o \leq h \leq h_m$ is imposed, a much greater improvement over the optimum steady-state cruise is obtained. This suggests that periodic cruise can greatly extend endurance when flight close to the earth's surface is required.

The research on periodic control of aircraft cruise is largely due to D. J. Lyons and will appear in [4], which is in final stages of preparation. This report should lead to several articles in the open literature.

Investigations in nonlinear systems theory have focussed on minimal order realizations for 2-power input/output maps of the form

$$y(t) = \int_0^\infty k(\tau_1, \tau_2)\, u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2.$$

A solution to this problem was presented in [5] for the case where k is continuous. Two extensions were obtained this year: the inclusion of impulsive terms in k, the treatment of the corresponding discrete-time problem. The minimal realizations obtained in these extensions have a particularly simple form. Do all other minimal realizations have a relation to these simple realizations? This question was answered affirmatively (for k continuous) in [6]. Moreover, the ideas extend to impulsive k and the discrete-time case. The consequence is a complete theory for the minimal realization of 2-power input/output maps. A paper which includes all of these results [7] is in preparation.

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